Algebra 2 Segment 2 – Lesson Summary Notes

For each lesson:

- ✓ Read through the LESSON SUMMARY which is located.
- ✓ Read and work through every page in the LESSON.
- ✓ Try each PRACTICE problem and write down the examples on the back of your lesson summary with the worked out solutions or on loose leaf paper to put in your MATH BINDER.
- ✓ Read the directions carefully for the assignment and submit the lesson ASSESSMENT.
- ✓ If you need to RESUBMIT (you see a BLUE ARROW) an assignment, make sure to view the feedback on the assignment by clicking on the assignment in your GRADES before resubmitting.

For more resources go to: http://eschoolmath.weebly.com/algebra-2.html

06.01 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- What methods can be used to solve a system of equations?
- What type of solutions do the methods for solving systems of equations find?

General Steps for Solving Systems of Equations by Elimination

1. Identify/Create opposite coefficients.
2. Add the equations vertically.
3. Simplify and solve for the first variable.
4. Substitute and solve. Substitute the value of the first variable into one of the original equations and solve for the second variable.

When both variables are eliminated and you are left with a …	
True Equation	**False Equation**
Final answer: Infinitely Many Solutions	Final answer: No Solution
The two equations graph the *same line*.	The two equations graph *parallel lines*.

Steps for Using Substitution to Solve a System of Equations

Step 1. Isolate one variable of one equation.
Choose an equation and solve for one of the variables.
Step 2. Substitute and solve for one variable.
Substitute the expression for the isolated variable into the other equation.
Solve the new equation for the variable.
Step 3. Substitute and solve for the other variable.
Substitute the value from the first variable into one of the original equations and solve.

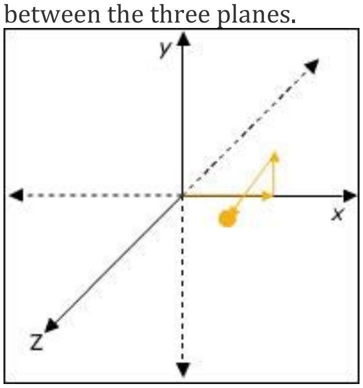
When the variable disappears and you are left with a …	
True Equation	**False Equation**
Final answer: Infinitely Many Solutions	Final answer: No Solution
The two equations graph the *same line*.	The two equations graph *parallel lines*.

Steps for Using the Elimination Method to Solve Systems of Equations with Three Variables

1. **Step 1: Identify or Create opposite coefficients**
   Identify or create opposite coefficients in two of the equations and add them vertically.
2. **Step 2: Identify or Create opposite coefficients again!**
   Identify or create opposite coefficients in two different equations. Add them vertically to eliminate the **same**variable.
3. **Step 3: Solve the New System**
   Solve the new system of equations with two variables.
4. **Step 4: Substitute and Solve**
   Substitute the values of the two variables into one of the original equations and solve for the last variable. Write the solution as an ordered triple.

Steps for Using the Substitution Method to Solve Systems of Equations with Three Variables

1. **Step 1: Pick and Solve.**
   Pick an equation and solve for one of the variables.
2. **Step 2: Substitute and Simplify.**
   Substitute the expression for the isolated variable into **both** of the other equations and simplify.
3. **Step 3: Solve the New System**
   Solve the new system of equations with two variables.
4. **Step 4: Substitute and Solve.**
   Substitute the values of the two variables into one of the original equations and solve for the last variable. Write the solution as an ordered triple.
5. The graphical representation of the solution to a system of equations with three variables is the point of intersection between the three planes.



06.02 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential question listed below.

- How can elimination and substitution adapt to solve a system with non-linear equations?

Non-Linear Systems of Equations

Non-linear systems of equations consist of two or more equations, at least one of which is of degree 2 or higher. These systems may be solved graphically by finding the point(s) of intersection or algebraically using the substitution or elimination method.

Substitution

1. Substitute one expression from the system into the other equation.
2. Solve for the first variable.
   - Combine like terms.
   - Factor or use the quadratic formula.
   - Solve.
3. Solve for the second variable.
4. Check your work.

Elimination

1. Isolate the y variable in both equations.
2. Create opposite coefficients and add the equations vertically.
3. Solve for the first variable.
   - Factor or use the quadratic formula.
   - Solve.
4. Solve for the second variable.
5. Check your work.

The solution(s) to non-linear systems of equations may be found by graphing each equation and finding the point or points of intersection.

06.03 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How can graphing methods be used to solve a system of linear equations?
- How can multiple representations of a system be graphed?

A system of equations is defined as two or more equations that use the same variables. The solution to these equations, if one exists, is the point(s) of intersection between the lines.

To find the solution to a system of two equations by graphing:

- Put the equations in slope-intercept form, if necessary.
- Graph both equations on the same coordinate plane.
  - Plot the y-intercept
  - From the y-intercept, use the slope to find a second point on the line. Remember, rise over run!
  - Connect the points to graph the line.
- Identify the point of intersection. The point of intersection is the solution to the system.

Equations with variables on each side can be separated into a system of equations.

- For example $4x + 2 = 5x - 1$ can be rewritten as:

$y = 4x + 2$

$y = 5x - 1$

- Solutions to the system of equations are also solutions to the original equation.

Equations can be represented in four ways:

- Verbally (like word problems)
- Algebraically
- In a graph
- In a table

Systems of linear equations have:

- One solution if they intersect.
- No solutions if they are parallel lines.
- Infinite solutions if they are the same line.

06.05 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How can graphing be applied to solving systems of non-linear equations?
- How can constraints be used to model a real-world situation?

A system of equations is defined as two or more equations that use the same variables. The solution to these equations, if one exists, is the point(s) of intersection between the lines.

To find the solution to a system of two equations by graphing:

- Graph both equations on the same coordinate plane. - Graphing by hand may require the use of a table to plot the points.
- Identify the point of intersection. The point of intersection is the solution to the system.
- Equations with variables on each side can be separated into a system of equations.
- For example $4x^2 + 2 = 5x^3 - 1$ can be rewritten as:

$$y = 4x^2 + 2$$

$$y = 5x^3 - 1$$

- Solutions to the system of equations are also solutions to the original equation.

Equations can be represented in four ways:

- Verbally (like word problems)
- Algebraically
- In a graph
- In a table

Constraints of a real-world situation can be used to find and solve a system of equations.

07.01 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How is an exponential function created?
- How can the properties of exponents help solve exponential equations?
- What are some real-world applications for exponential functions?

Solving Exponential Equations

- Create equal bases on both sides of the equation.
- Solve for the variable after eliminating the identical bases.
- Check your work.

Exponential Growth and Decay Functions

- An **exponential growth function** is a function where the amount of money or of a substance *increases* exponentially as time passes. The base of the exponent must be greater than 1.
- An **exponential decay function** is a function where the amount of money or of a substance *decreases* exponentially as time passes. The base of the exponent must be between 0 and 1.
- The constant number e is used in a variety of science and mathematical applications where an amount grows or decays consistently over time. The constant e is equivalent to 2.71828....

Important Exponential Functions

Compound interest is A=$P(1+\frac{r}{n})^{nt}$

- The variable A represents the amount after t years.
- The principal, or the amount at the beginning, is P.
- The interest rate, or rate of change, is r, written in decimal format.
- The number of times interest is earned, or compounded, on the account each year is n.

Continuous interest is A=Pe^{rt}.

- The variable A represents the amount after t years.
- The principal, or the amount at the beginning, is P.
- The interest rate, or rate of change, is r, written in decimal format.

07.02 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential question listed below.

- How are exponential functions related to logarithmic functions?
- How can a logarithmic function be created?
- How can the properties of exponents help solve logarithmic equations?

To solve a logarithmic equation or evaluate a logarithmic expression, follow the steps listed below.

1. Rewrite the logarithm in exponential form.
2. Solve for the variable.
3. Check your work.

Logarithmic Functions

- A **logarithmic function** is the inverse of an exponential function. It may be written in the format $x = \log_b y$ where x is the power, b is the base, and y is the argument.
- Logarithms of base 10 are written without the base as $x = \log y$. Logarithms of base *e*, where *e* is equal to 2.718218…are written as $x = \ln y$.
- Logarithmic equations can be converted into exponential equations, and vice versa, using the following format.
  $y = b^x \leftrightarrow x = \log_b y$

07.03 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How are exponential functions related to logarithmic functions?
- How can a logarithmic function be created?
- How can the properties of exponents help solve logarithmic equations?

Change of Base Formula

The **Change of Base Formula** is used to evaluate logarithmic expressions whose base is not equal to 10. You will see the application of this formula when graphing logarithmic functions. To utilize the formula, the common log of the argument is divided by the common log of the base.

Change of Base Formula

$$\log_b y = \frac{\log y}{\log b}$$

Equality Property

Given an equation where two logarithmic expressions, with identical bases, are equal to each other, the arguments of those expressions are also equal.

If $\log_4 x = \log_4 y$,
then $x = y$.

Product Property

When two or more logarithmic expressions with the same base are added, the arguments may be multiplied.

$\log_7 3 + \log_7 5$
$\log_7 (3)(5)$
$\log_7 15$

Quotient Property

When two or more logarithmic expressions with the same base are subtracted, the arguments may be divided.

$\log_7 54 - \log_7 9$
$\log_7 \frac{54}{9}$
$\log_7 6$

Power Property

A coefficient on a logarithmic term can be moved to the exponent of its argument and an exponent can become a coefficient.

$4 \log_7 3$
$\log_7 3^4$
$\log_7 81$

Logarithmic Equations with Constants

To solve a **logarithmic equation using constants**, isolate the constant terms on one side of the equation. Isolate the logarithmic terms on the other side. Then, convert the logarithmic equation into an exponential equation.

$\log_7 x = 3$
$7^3 = x$
$343 = x$

07.04 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How can exponential equations with unequal bases be solved?
- How do the various parts of an exponential equation affect the entire equation?

Solving Exponential Equations with Unequal Bases

- Rewrite the exponential equation as a logarithmic equation.
- Solve for the variable.
- Check your work.

Finding Rates of Growth or Decay for Different Time Intervals

- Given an exponential function $A(t) = P(1 + r)^t$, the exponent can be modified to find the rate of growth or decay for a different time interval.
- Use $A = P(1 + r^{\frac{1}{c}})^{c*t}$
  where c is the number of times more the growth or decay will be calculated.

07.06 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

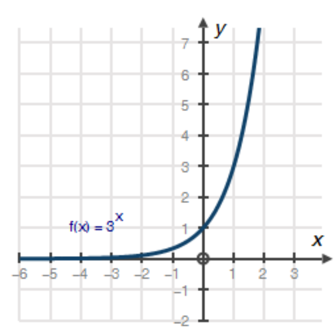
- Why does the average rate of change vary on an exponential function?
- What effects does adding a constant have to an exponential function?
- How can exponential functions be utilized to fit existing data?

Graphing Exponential Functions

An **exponential function** is a function where the variable is located in the exponent and the base of the function is limited to positive real numbers not equal to 1.

An exponential function may be graphed using technology such as a graphing software program or graphing calculator.

1. Open the graphing software program/calculator of your choice.
2. Set up the coordinate plane by ensuring the grid and axes are showing.
3. Enter the equation f(x) = 3^x. If the exponent contains more than just the variable (e.g., x + 1), you **must** include parentheses around the expression in the exponent or an incorrect function will be graphed.
4. After entering the function, a graph will appear.



General exponential functions have a domain of all real numbers, a range of $y > 0$, and a horizontal asymptote at $y = 0$.

Key Features of Exponential Functions

- The y-intercept
- End behavior
- Average rate of change

Fitting Functions to Data

- A function is said to be a "best fit" if it is the function that passes closest to the given data points.

07.07 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can logarithms be graphed with different bases?
- How do transformations affect the logarithmic graph?

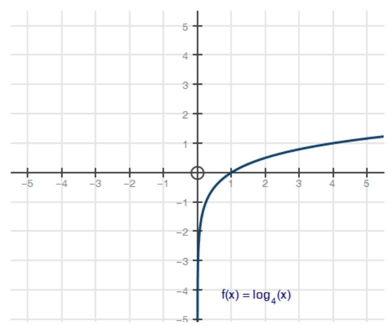
Graphing Logarithmic Functions

A **logarithmic function** is the inverse of an exponential function. The base of the function is a positive real number not equal to 1.

Graphing a logarithmic function may be done using graphing technology. It may be necessary to use the Change of Base Formula to convert a base unequal to 10 to the common log of base 10 if you are using a different graphing program.

$$f(x) = \log_4 x \rightarrow f(x) = \frac{\log x}{\log 4}$$

1. Open the graphing program/calculator of your choice.
2. Set up the coordinate plane. Ensure the grid and axes are showing.
3. Enter the equation.
4. After entering the function, the graph will appear



General logarithmic functions have a domain of $x > 0$, a range of all real numbers and a vertical asymptote at $x = 0$.

07.08 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How can exponential and logarithmic functions be combined?
- What effect does adding a linear function to an exponential or logarithmic function have on the graph?
- In what ways do inverse functions relate to exponential functions and logarithmic functions?

Arithmetically Combined Functions

Functions can be combined using basic arithmetic operations. Make sure when adding or subtracting that only like terms are combined. Distribute completely when multiplying. Factor the numerator and denominator to simplify the division.

$f(x) + g(x)$ $\qquad$ $f(x) \bullet g(x)$

$f(x) - g(x)$ $\qquad$ $\frac{f(x)}{g(x)}$

Special Cases

Logarithmic and exponential functions cannot be simplified after combination, except in special cases.

- Adding logarithms with the same base. Use the Product Property of Logs.
- Subtracting logarithms with the same base. Use the Quotient Property of Logs.
- Dividing logarithms with the same base. Use the Base Change Formula to simplify.

Inverse Functions

Exponential functions are the inverse of logarithmic functions, and logarithmic functions are the inverse of exponential functions.

$$b^x = a \leftrightarrow \log_b a = x$$

08.01 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can an expression or process be determined for an arithmetic sequence?
- What types of functions combine to create an explicit formula for arithmetic sequences?
- What possible restrictions exist on domains and ranges of arithmetic sequences?

Arithmetic Sequences

A sequence is a list of numbers, also called terms, separated by commas. An arithmetic sequence is a list of numbers where the difference between each term is constant.

A specified term of an arithmetic sequence may be found by the formula $a_n = a_1 + (n - 1)d$ where a_1 is the first term of the sequence, n is the position number of a given term, d is the common difference between the terms, and a_n is the value of the term in the given position number.

Arithmetic Sequence Formula:

$$a_n = a_1 + (n - 1)d$$

Recursive vs. Explicit

- A recursive process requires the calculation of each term as each term relies on the term before it.
- An explicit equation allows the calculation of any term in the sequence.

Recursive

$$a_n = a_{n-1} + d$$

Explicit

$$a_n = a_1 + d(n - 1)$$

Restrictions

- The domain is restricted to the integers greater than or equal to one. These are the counting or “natural” numbers.
- The range is unrestricted.

08.02 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How do arithmetic sequences and series differ?
- What important information must be included to find a sum of a series?

Arithmetic Series

An arithmetic series is a list of numbers, separated by a common difference, that are being added.

The sum of an arithmetic series may be found by the formula $S_n = \frac{n}{2}(a_1 + a_n)$ or $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$ where a_1 is the first term of the series, n is the number of terms in the series, d is the common difference between the terms, a_n is the final term in the series and S_n is the sum of the series.

Arithmetic Series Formula:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

08.03 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can an expression or process be determined for a geometric sequence?
- What functions combine to create an explicit formula for geometric sequences?
- What possible restrictions exist on domains and ranges of geometric sequences?

Geometric Sequences

A geometric sequence is a list of numbers where the ratio between each term is consistent.

A specified term of a geometric sequence may be found by the formula $a_n = a_1 r^{(n-1)}$ where a_1 is the first term of the sequence, n is the position number of a given term, r is the common ratio between the terms, and a_n is the value of the term in the given position number.

Geometric Sequence Formula:

$$a_n = a_1 r^{(n-1)}$$

Recursive vs. Explicit

- A recursive process requires the calculation of each term as each term relies on the term before it.
- An explicit equation allows the calculation of any term in the sequence.

Recursive	Explicit
$a_n = r(a_{n-1})$	$a_n = a_1(r)^{n-1}$

Restrictions

- The domain is restricted to the integers greater than or equal to one. These are the counting or “natural” numbers.

- The range is unrestricted.

A combination of functions

The explicit equation for geometric sequences is a combination of a constant function (a_1) multiplied by an exponential function (r^{n-1}).

08.04 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How do geometric sequences and series differ?
- What important information must be included to find a sum of a geometric series?
-

Geometric Series

A geometric series is a sequence of numbers, separated by a common ratio, that are being added.

The sum of a geometric series may be found by the formula $S_n = \frac{a_1 - a_1 r^n}{1-r}$ or $S_n = \frac{a_1 - a_n r}{1-r}$ where a_1 is the first term of the sequence, n is the number of terms in the series, r is the common ratio between the terms, a_n is the last term in the series and S_n is the sum of the series.

Geometric Series Formulas

$$S_n = \frac{a_1 - a_1 r^n}{1-r}$$

OR

$$S_n = \frac{a_1 - a_n r}{1-r}$$

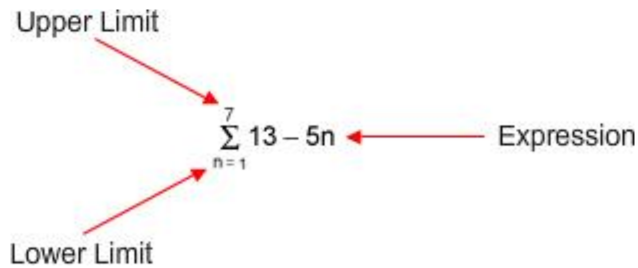
08.06 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below.

- How is sigma notation used to find the value of a series?
- What must be true to find the sum of a infinite sequence?

Sigma Notation

Sigma notation is characterized by the Greek letter for "S," Σ, which is sigma. In the notation, there exists a lower index, upper index, and an expression which explains an arithmetic sequence or series.



To find the expression representing an arithmetic sequence, substitute the first term and the common difference of the arithmetic sequence into the formula $a_n = a_1 + (n - 1)d$.

Arithmetic Sequence: 8, 3, −2 ...

$$a_n = a_1 + (n - 1)d$$
$$a_n = 8 + (n - 1)(-5)$$
$$a_n = 8 - 5n + 5$$
$$a_n = 13 - 5n$$

Geometric Sequence

When dealing with a geometric sequence (such as 2, 4, 8, ...) there is no simplification of the sequence's expression like you can with an arithmetic sequence. Find the first term and the common ratio and substitute them into the expression.

$$\frac{a_1 - a_1 r^n}{1 - r}$$

$$a_n = 2\,(2)^{n-1}$$

08.07 Lesson Summary

Infinite, Convergent, and Divergent Series

The partial sum of any infinite arithmetic or geometric series can be found by utilizing the appropriate summation formula:

$S_n = \frac{n}{2}(a_1 + a_n)$ for arithmetic series

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ for geometric series.

The sum of an infinite arithmetic series can never be found, since the sum is continuously increasing with the addition of each term. An infinite **divergent** geometric series is a series where the sum of terms does not approach a specific value. In a divergent geometric series, the absolute value of the common ratio is greater than 1.

$$\sum_{i=1}^{\infty} 4(-3)^{i-1} = 4 - 12 + 36 - 108 + \dots$$

However, the sum of the terms of an infinite **convergent** geometric series approaches a specific value. The absolute value of the common ratio of an infinite convergent geometric series is less than 1 but not equal to 0. The approximate sum of this type of series can be found using the formula

$$Si=\frac{a1}{1-r}$$

$$\sum_{i=1}^{\infty} 90\left(\frac{1}{10}\right)i-1=90+9+0.9+0.09+\ldots=99.99\ldots\approx 100$$

To help you remember which type of series may be found, refer to the following chart:

Sums of Arithmetic and Geometric Series Can the Sum Be Found?		
	Arithmetic Series	**Geometric Series**
Partial Sum	Yes	Yes
Infinite Sum	No	Divergent ($\lvert r\rvert > 1$) No Convergent ($\lvert r\rvert < 1, r \neq 0$) Yes

08.08 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can an expression or process be determined for a geometric sequence?
- What functions combine to create an explicit formula for geometric sequences?
- What possible restrictions exist on domains and ranges of geometric sequences?

Restrictions to Range and Domain

- The domain is restricted to all integers great than or equal to 1.
- The range has no restrictions unless the situation dictates that only certain values are acceptable.

Discrete Graphs

- The graphs of sequences and series are discrete, meaning they are only valid at specific points. They are not continuous lines or curves passing through all numbers.

Graphing Sequences

- To graph arithmetic sequences, use $an = a_1 + d(n - 1)$.
- To graph geometric sequences, use $an = a_1(r)n - 1$.

Graphing Series

- To graph arithmetic series, use $S_n = \frac{n}{2}(2a_1 + d(n - 1))$.
- To graph geometric series, use $S_n = \frac{a_1 - a_1 r^n}{1 - r}$

Average Rate of Change

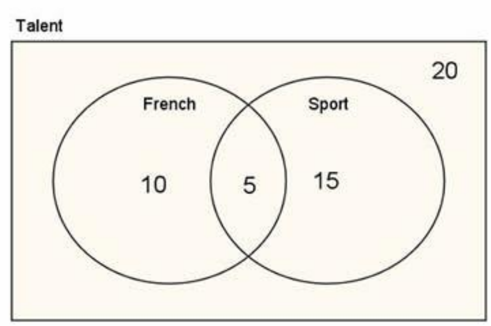
- Use the slope formula to find the average rate of change between points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

09.01 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to this essential question: How can you represent relationships between sets and subsets of a sample space?

The set of all outcomes of an experiment is called a **sample space.**
An **event** is a subset of a sample space.
A **subset** is a group of elements that is completely contained in the sample space.
The **empty set**, ∅, is a subset of every set. There are no elements in the empty set.
Venn diagrams and two-way tables can be used to organize categorical data.



	Plays a sport	Does *not* play a sport	Total
Speaks French	5	10	15
Does *not* speak French	15	20	35
Total	20	30	50

Let A = contestants that speak French
Let B = contestants that play a sport
The union of two sets *A* and *B* is the set that contains elements that are in *either* set *A or B* and is written as *A*∪ *B*. In this example, there are 30 students in A ∪ B.
The intersection of two sets *A* and *B* is the set that contains elements that are in *both* sets *A and B* and is written as *A ∩ B*. In this example, there are 5 students in A ∩ B.
The complement of set *A*, or *A'*, is the set of all elements that are in the sample space but not in set *A*. In this example, there are 40 students in A'.

Set A is contained in set B if all of the elements in set A are also in set B. This is written as $A \subseteq B$. But notice this cannot be done in the Talent example since there are students in set A not included in set B.

09.02 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How do you determine if two events are independent?
- How can we interpret the independent or conditional probability of two events?
- How can you construct two-way frequency tables of data?

The probability of an event, $P(E)$ = number of favorable outcomes / total number of outcomes in the sample space
Two events are independent if the outcome of the second event is not affected by knowing the outcome of the first event.

Two events A and B are independent if $P(A) \bullet P(B) = P(A \cap B)$.
Conditional probability is the probability of an event occurring given the occurrence of some other event.

The probability of event A occurring given that event B has occurred is $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Two-way frequency tables can be used to display categorical data.

09.03 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can you explain the concepts of conditional probability and independence in everyday situations?
- How can you find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model?
- How can you apply the Addition Rule and interpret the answer in terms of the model?

P(A | B) is not always the same as P(B | A).

When calculating conditional probability, P(A | B), start by identifying the new, reduced sample space which is the total number of outcomes in event B. Then determine the number of outcomes in $A \cap B$. Now $P(A \mid B) = \frac{\text{number of outcomes in } B \cap A}{\text{number of outcomes in } B}$
.

Example: The two-way table below shows the different colors of birds and what they like to eat.

	Red	**Blue**	**Total**
Seeds	3	4	7
Insects	1	2	3

	Red	Blue	Total
Total	4	6	10

Let A = red birds and let B = seeds.

Using the formula P(A | B) = $\frac{\text{\# of outcomes in } B \cap A}{\text{\# of outcomes in } B}$ to determine the probability that a bird is red, given that it eats seeds.

P(A | B) = $\frac{3}{7}$

Using the Fraction

Use P(A | B) = $\frac{P(A \cap B)}{P(B)}$ to determine the probability that a bird is red, given that it eats seeds.

P(A | B) = $\frac{\frac{3}{10}}{\frac{7}{10}}$ which simplifies to $\frac{3}{7}$.

P(A ∪ B) is the probability that *either* event A *or* event B happens.
P(A ∩ B) is the probability that *both* A *and* B happen.
Mutually exclusive events are events that can never happen at the same time.

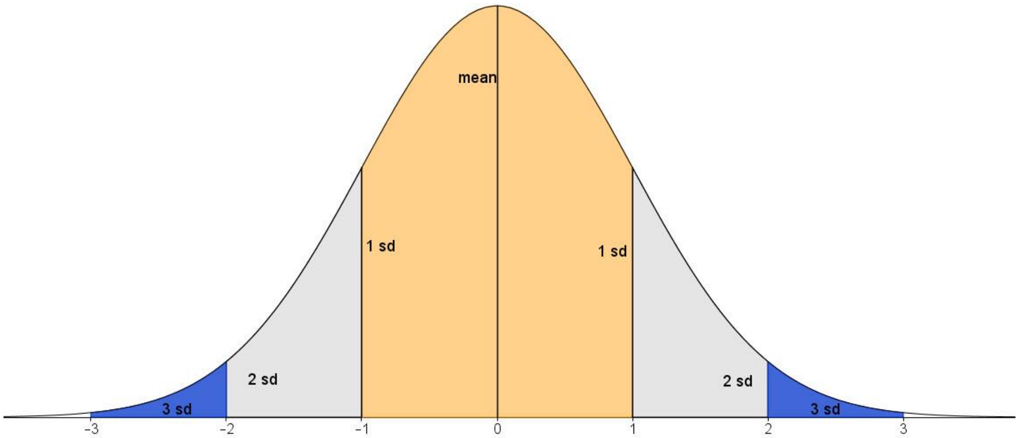
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

With mutually exclusive events, $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.

09.05 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential question listed below.

How can you use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages?



Normal distributions are only appropriate for unimodal, symmetric data sets.

Empirical rule: 68–95–99
Approximately 68% of the data is within 1 standard deviation of the mean.
Approximately 95% of the data is within 2 standard deviations of the mean.
Approximately 99% of the data is within 3 standard deviations of the mean.
Standardizing a raw score (finding a z–score)

$$z = \frac{x - \mu}{\sigma}$$

Use the z–score to look up the probability on the table. The probability given in the table is always the area to the left of the z–score.

09.06 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can you use statistics to make inferences about a population based on a random sample from that population?
- How can you decide if a specified model is consistent with results from a data-generating process?
- How can you use surveys, experiments, and observational studies to make inferences about a population?
- How can you define appropriate quantities for descriptive modeling?

An experiment is a study which involves separating the sample into two or more groups, applying a treatment to one of the groups, and leaving the other group untreated. Experiments can be used to find cause and effect relationships.

An observational study is a study in which control and trial groups are naturally occurring, and observations are made about something of interest to the researcher. Observational studies can be used to find possible relationships.

A sample survey is a process for collecting data from a sample by asking questions. A survey cannot be used to show a cause and effect relationship. A survey is typically used to describe something about a population that is being studied.

Simulations can be used to compare theoretical and empirical probabilities and determine how plausible the results are.

In evaluating reports, try to answer the following questions:

- What is the purpose of the study?
- How was the data collected?

- Are the data collection methods appropriate for the type of study?
- Was there any potential bias? If so, how was it reduced?
- Who were the participants and how were they chosen?
- Were the correct statistical tests used for the data?
- Was the data presented?
- What are the results?
- Are the conclusions appropriate?

In an observational study or survey, the results can estimate population proportions or population means, or suggest there may be a relationship between variables. A cause and effect relationship cannot be established through observational studies or surveys.

09.08 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can we use data from a randomized experiment to compare two treatments?
- How can we use simulations to decide if differences between parameters are significant?
- How can we evaluate reports based on data?
- How can we define appropriate quantities for the purpose of descriptive modeling?

An experiment should consist of the following parts:

- Introduction and research question
- Methodology – including how the sample will be chosen, randomization, and any decisions that will be made during the study
- Results should include all data that was collected and any statistical tests
- Discussion – including any conclusions, benefits, and suggestions for further study.

Bias is introduced through design and one way to reduce possible bias is through randomization of the control and treatment groups.

To test whether a difference between treatment and control groups exists, the data can be re-randomized many times to test the probability that the difference of means that was found in the study is likely to happen by chance.

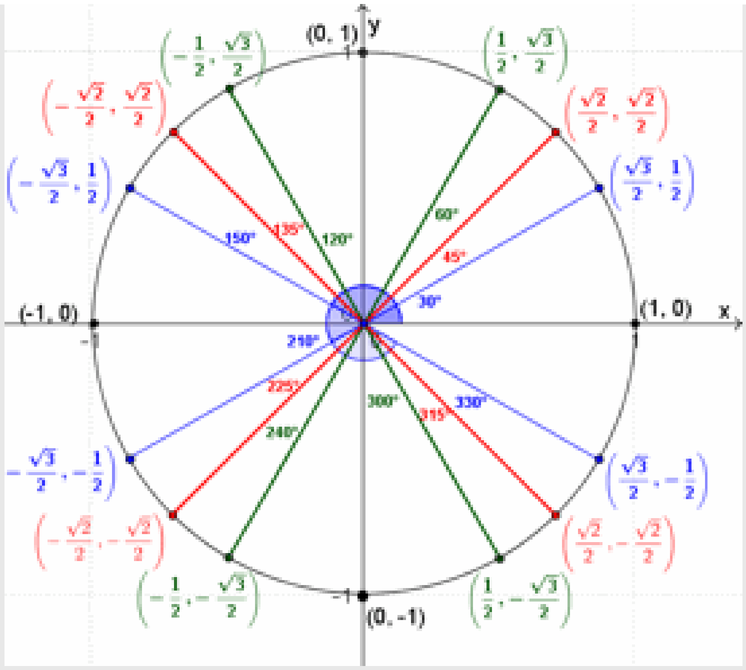
10.01 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below:

- How can the radian measure of an angle determine the arc length on the unit circle?
- How can the unit circle be used to apply trigonometric functions to all real numbers?

The Unit Circle

The unit circle is a circle centered at the origin of the coordinate plane with a radius of 1. The unit circle uses central angles and coordinate points of intersection to evaluate trigonometric functions. Those coordinate points and their associated angles are found in the image below:



SOH-CAH-TOA

The mnemonic SOH-CAH-TOA can be used to remember the trigonometric ratios for sine, cosine, and tangent. If you use a triangle with a vertex as a central angle on the unit circle, the ratio can also be expressed in terms of x and y. The symbol theta (θ) is used to represent the measure of an angle in standard position.

Trig Function	sin θ	cos θ	tan θ
Ratio	$\frac{y}{r}$	$\frac{x}{r}$	$\frac{y}{x}$

A radian is the measure of a central angle that intercepts an arc equal in length to a radius of the circle.

Degrees to Radians

$$\text{Radians} = \text{Degrees} \bullet \frac{\pi}{180^\circ}$$

Radians to Degrees

$$\text{Degrees} = \text{Radians} \bullet \frac{180^\circ}{\pi}$$

When the measure of an angle θ is in radians and *r* is the radius, the length *s* of the intercepted arc is s = rθ.

arc length: s=rθ

(θ must be in radians)

10.02 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential question listed below and make note of the key ideas in the lesson.

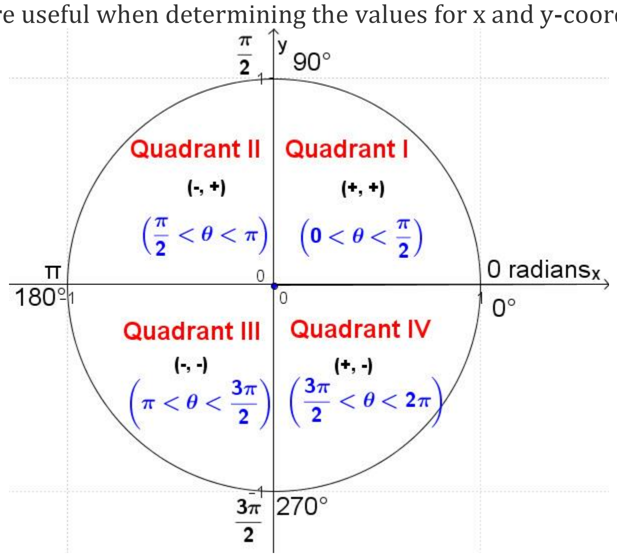
How can the unit circle and radian measures be used to apply trigonometric functions to all real numbers?

You can find trigonometric values and the ordered pairs of points outside the unit circle using the trig ratios.

Trig Function	Sin Θ	Cos Θ	Tan Θ
Ratio	$\frac{y}{r}$	$\frac{x}{r}$	$\frac{y}{x}$ where x ≠ 0

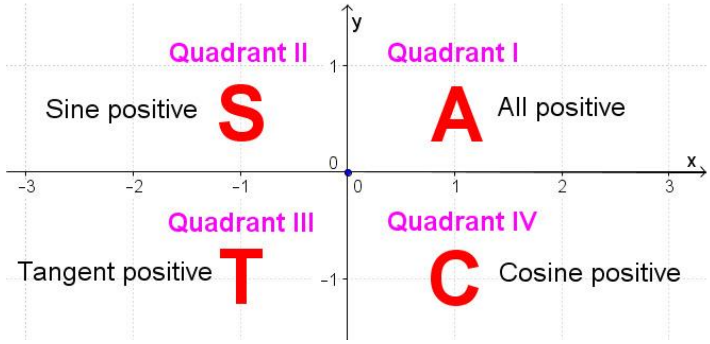
$$r = \sqrt{x^2 + y^2}$$

The sign values for each quadrant are useful when determining the values for x and y-coordinate points.



- A **reference angle**, indicated by the symbol Θ', is the acute angle formed by the terminal side of the angle and the horizontal axis.

- Reference angles can be used to determine the value of trigonometric functions for any angle. The sign of the function is based on the quadrant that contains the terminal side. Use the diagram to help you remember which trig function is positive in which quadrant.



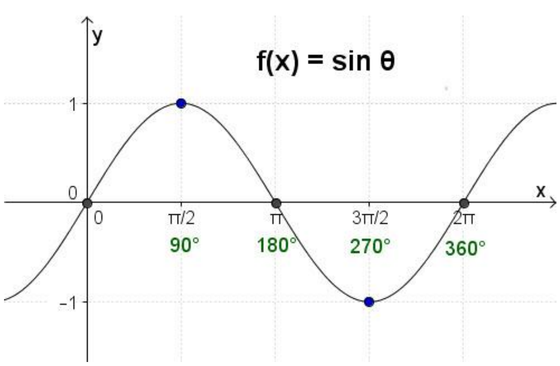
10.04 Lesson Summary

To achieve mastery of this lesson, make sure that you develop responses to the essential questions listed below and make note of the key ideas in the lesson:

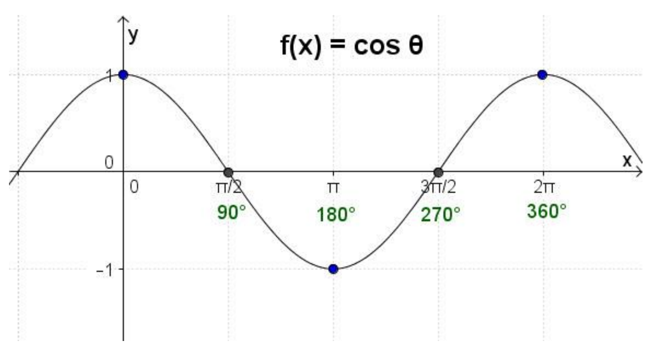
- How are trigonometric functions graphed to show the period, midline, and amplitude?
- How can a trigonometric function be chosen to model periodic phenomena with specified amplitude, frequency, and midline?
- How does replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative) effect the graph?
- How does a function model the relationship between two quantities?
- What is the rate of change of a function over a specified interval?
- How can a function fitted to data be used to solve problems?

Sine and cosine curves can be expressed using the following standard functions:

$$f(x) = a\sin(bx - c) + d$$



$$f(x) = a\cos(bx - c) + d$$



Transforming the trigonometric functions is similar to transforming other functions. Follow the guidelines below for sine and cosine:

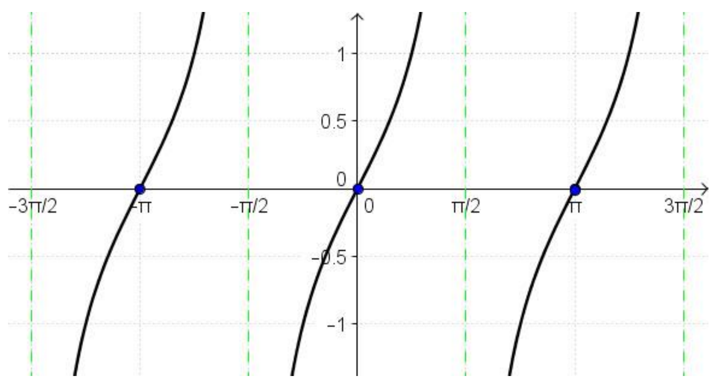
- **Amplitude**: The value of **|a|** is the **amplitude** of the curve and determines the *vertical* stretching or compressing of the curve.

- **Period**: The value of **b** determines the **period** of the graph and determines the *horizontal* stretching or compressing of the curve. For positive values of **b**, values less than 1 stretch the period beyond 2π, and values greater than 1 compress the period to less than 2π. Period = $\frac{2\pi}{|b|}$

- **Phase Shift**: The value of **c** determines a *horizontal* shift of the curve along the x-axis. The length of the shift is determined by $\frac{c}{b}$, also known as the **phase shift.** The endpoints of one interval, or cycle, can be determined by solving the equations bx – c = 0 and bx – c = 2π

- **Vertical Shift**: The value of **d** is the *vertical* shift of the trigonometric function along the y-axis. This shift creates a new, imaginary x-axis for the graph to be centered around. It is called the **midline.**

Tangent is expressed using the standard function: **f(x) = a tan(bx – c)**



Use the following guidelines for transforming tangent functions:

- Period = $\frac{\pi}{|b|}$
- Phase Shift/Starting Point: Solve bx –– c = 0
- Spacing for Intercepts and Asymptotes: Add $\frac{\pi}{2b}$ to each point, beginning with the starting point.
- Find the y-intercept by evaluating f(0).

The rate of change of a trigonometric function can vary greatly depending on the point at which the rate of change is measured. Use the equation for slope to find the rate of change over a specified interval: m = $\frac{y_2 - y_1}{x_2 - x_1}$

10.05 Lesson Summary

A trigonometric identity is a statement that is true for all values of Θ except for those values that make the equation undefined. One of the most useful trigonometric identities is the Pythagorean Identity.

Pythagorean Identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

The identity $\tan \theta = \frac{\sin\theta}{\cos\theta}$ along with the Pythagorean Identity can be used to simplify, solve, and prove expressions and equations.

10.06 Lesson Summary

- During Algebra 2, you explored linear, quadratic, cubic, polynomial, rational, exponential, logarithmic, and trigonometric functions.
- There are many ways to represent functions—a table, graph, description, an equation.
- These functions can be examined individually and compared to each other through their commonalities—x-intercepts, y-intercepts, rate of change, maximums, and minimums.
- It is important to note the intervals on which the functions are being compared.